Spruce Budworm Model and Predator / Prey Models

By Joseph Pousada May, 2 2013

Populations models are an important for us to learn about so we can manage our ecosystems and biodiversity. The topic of predator - prey studies is as immense as our biodiversity. I am going to explore population models and how they relate to outbreaks as well as predator-prey relationships. The topic is a very broad but here we will explore the spruce budworm model, explore in general population models as well as predator-prey models and see if there is an underlying connection and will look at the spruce tree to see if we can enhance the spruce budworm model.

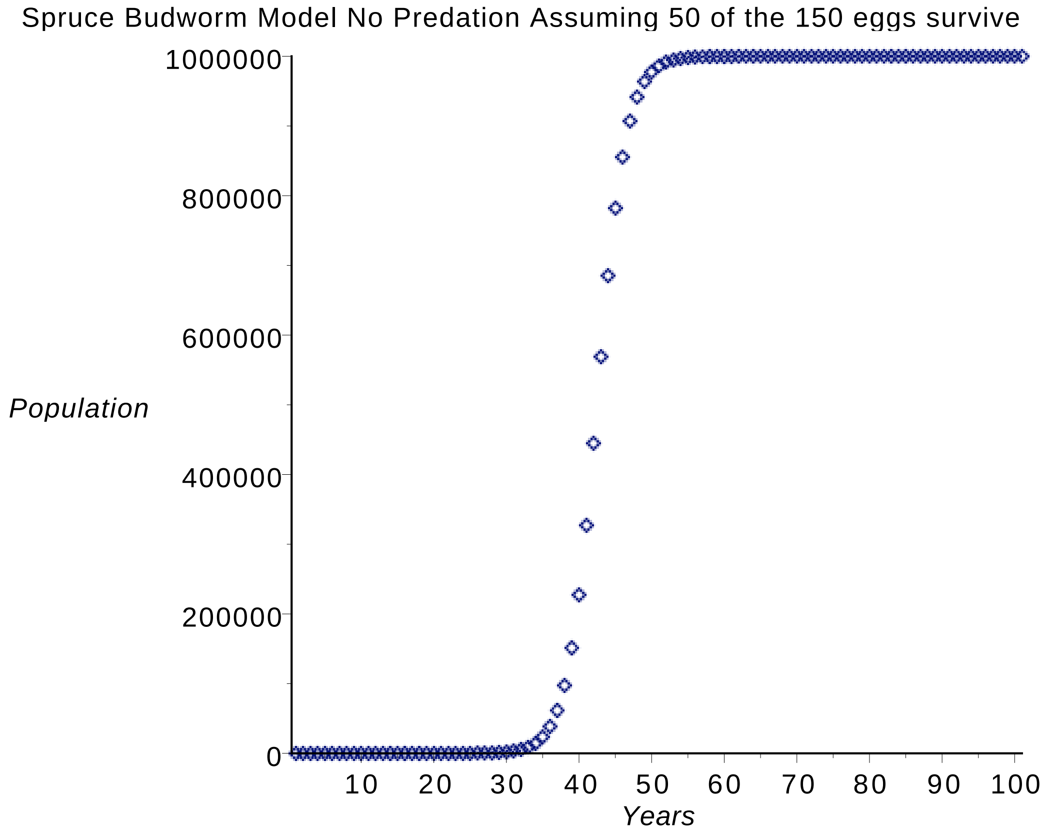
The Spruce Budworm Model

The spruce budworm is native to North America so it is not an invasive species without any known local predators. It feeds off of various trees which includes the various spruce trees in the northern latitudes of North America. The species has had outbreaks in various regions at various times in recorded history. Government agencies are interested in why the species goes through outbreaks primarily because there is an economic cost that results from the damage to the trees.

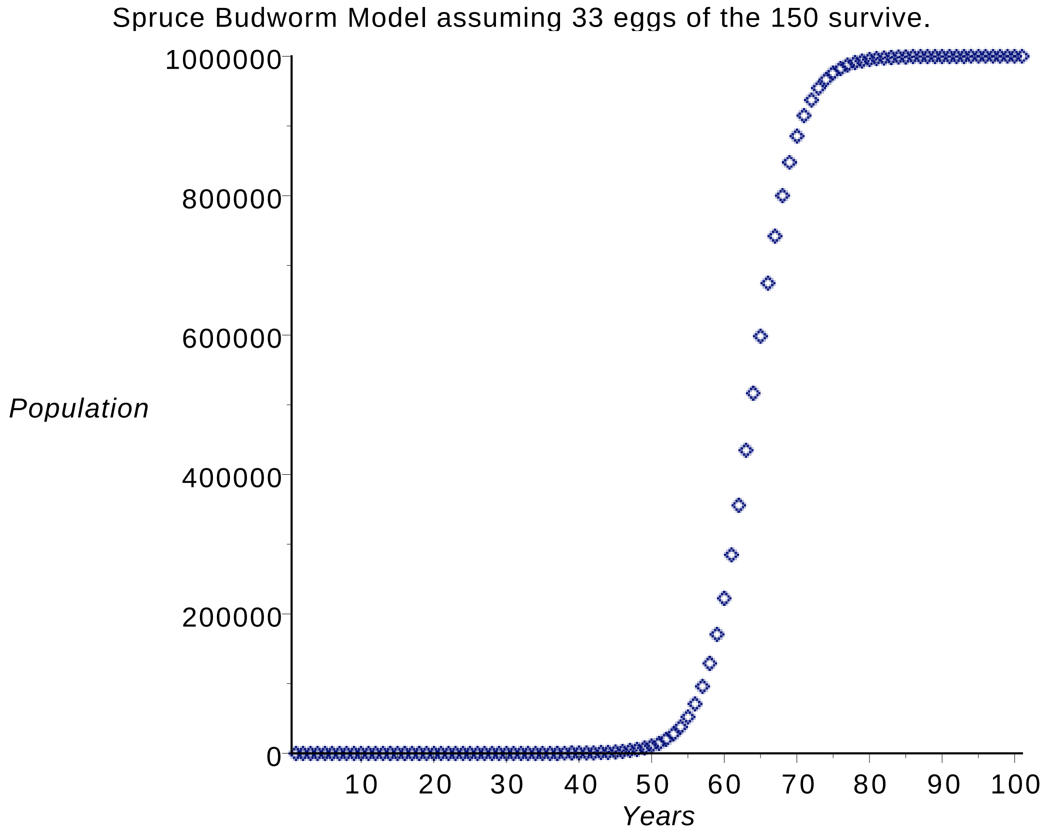
The basic model for he Spruce Budworm Model without predation is:

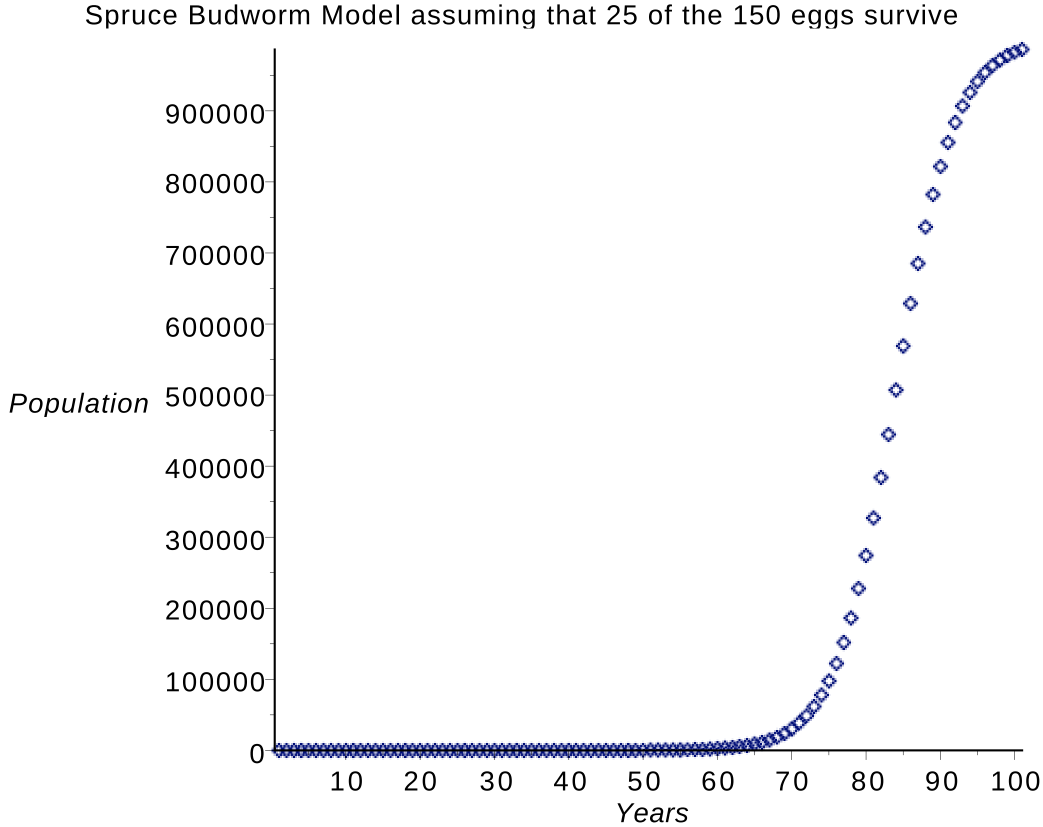
" $\frac{dN}{dt} = r_B N\left(1 - \frac{N}{K_B}\right)$ " (White & Huang) with $r_B =$ the intrinsic birthrate of the Budworms and with K_B the "carrying capacity"(White & Huang) of the geographic region.

The US Department of Agriculture and Forest Service states that the spruce budworm will lay approximately 150 eggs. They state that while in the eggs it is very resilient to low temperatures but that in May and June the larvae will leave the egg and feed on the trees. During these early states it is vulnerable to low temperature changes as well as other factors such as microbial diseases. We will look to see how this model works and assume an initial population of 1000 and that this is a new island for the species off the cost of British Columbia and that it has a "carrying capacity" of 1MM. If we assume that approximately 50% of all spruce budworms are female and that only approximately 50 of the 150 eggs survive we can look to see how this outbreak occurs below:



Even if we want to consider years which there may have been somewhat unfavourable weather conditions (lets say 33 eggs of the 150 eggs survive) to unfavourable weather conditions (lets say only 25 of the 150 eggs survive) we have the following models:


Spruce Budworm Model assuming 33 eggs of the 150 survive.
1000000
800000
600000
400000
200000
0
Population
10
20
30
40
50
60
70
80
90
100
Years



Interestingly enough we see that while the model is shifted to take longer for an outbreak to occur if conditions are not as favourable it does not prevent the outbreak from occurring outright. It is also interesting to note that it increases as a modest rate then hits a critical point where there is an out break and then levels off where it reaches the "carrying capacity".

So what happens when we introduce predation. The model given for the spruce budworm with predation is:

" $\frac{dB}{dt} = rB\left(1 - \frac{B}{K}\right) - \beta\left(\frac{B^2}{\alpha^2 + B^2}\right)$ " (Moody's, pg278)

The predation assumed in the model was that of the birds eating the spruce budworms. One way we could improve this model would be to take a sample count of not only the bird predators in a given area, but a sample count of spiders and other small creatures that prey on the Spruce Budworm and incorporate them into the model as well.One easy way is to modify the constant for the bird population to be (Bird+Spider+Others) and wrap them up into one constant.

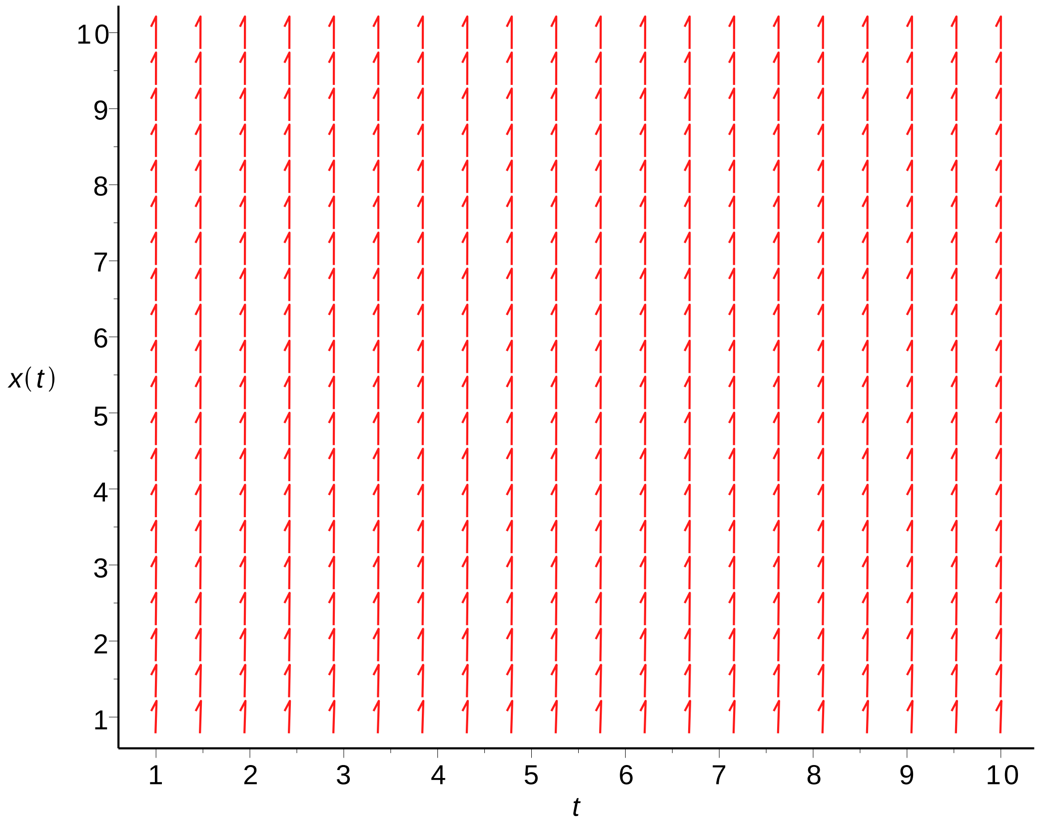
After transforming the model to a "dimensionless coordinate system"(Moody's, pg 279), the model is now represented as:

"$\frac{dx}{d\Gamma} = \left(Rx\left(1 - \frac{x}{Q}\right) - \left(\frac{x^2}{1+x^2}\right)\right)$ "(Moody's, pg 279)

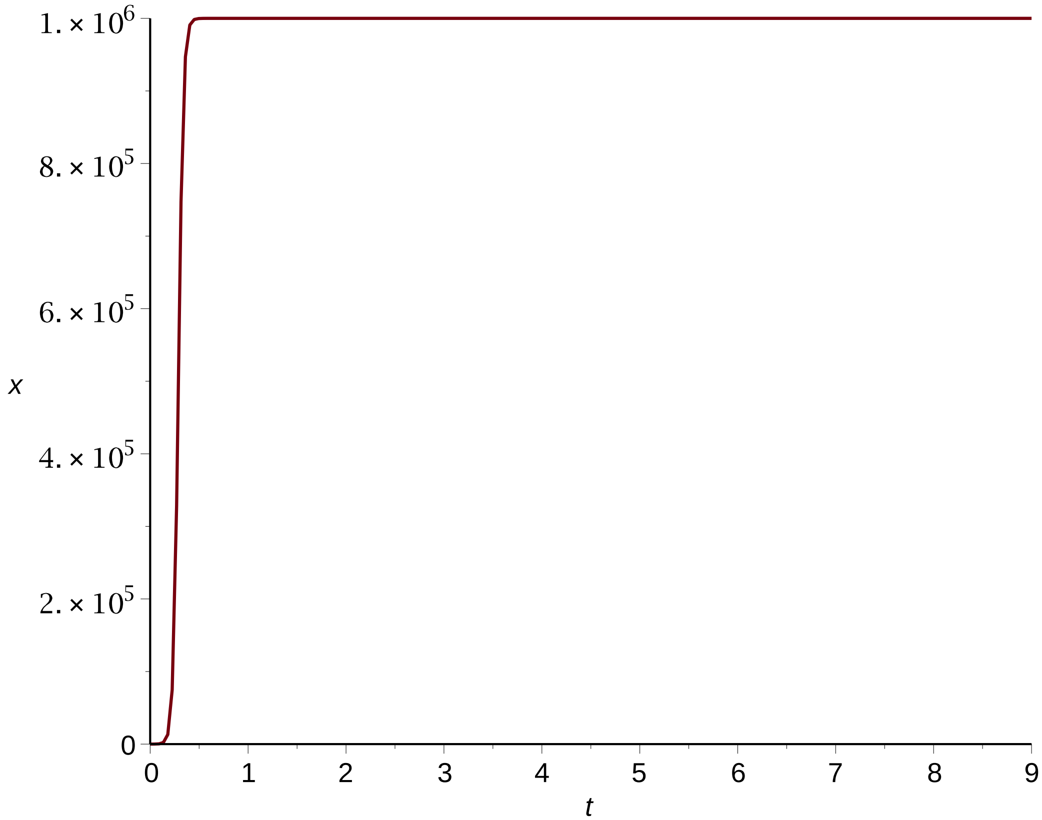
So if we let R=40 and Q=1MM and let maple derive the solution with the initial condition that the initial population is 1000 we get the following:

$$x(t) = RootOf\left(t + \int_{\_b}^{\_Z} \frac{25000\,(1+\_a^2)}{\_a\,(-1000000 - 1000000\,\_a^2 + 25001\,\_a + \_a^3)}\, d\_a - \left(\int_{\_b}^{1000} \frac{25000\,(1+\_a^2)}{\_a\,(-1000000 - 1000000\,\_a^2 + 25001\,\_a + \_a^3)}\, d\_a \right)\right)$$

The direction field for this is:



And if we plot the ode assuming we start with 10 spruce budworms we get:



One the outbreak takes place the upward pressure is high and only limited by the capacity of the island and the few predators that are on the island become irrelevant. What happens in real life though is that the spruce trees become damaged or die and therefore the carrying capacity of this island goes down and the population of the spruce budworm will follow.

Population Models

It turns out that the Spruce Budworm model is not a unique model and is the standard model for population dynamics used with the general logistic equation being:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)$$

which when we solve for the differential equation in maple we get:

$$x(t) = \frac{K}{1 + e^{-rt}\_C1\,K}$$

While this model does not include the predation it still very important and is useful for us to get an intuitive perspective in why this model is describing the behaviour of population dynamics. When plants that lived near the shores moved to be able to live on land this was a new species starting out on a new land mass with no competition/predators. The "carrying capacity" can be viewed as the land mass itself. Whether it be a large continent or a smaller land mass here is an upperlimit to how many species can thrive on the land mass.

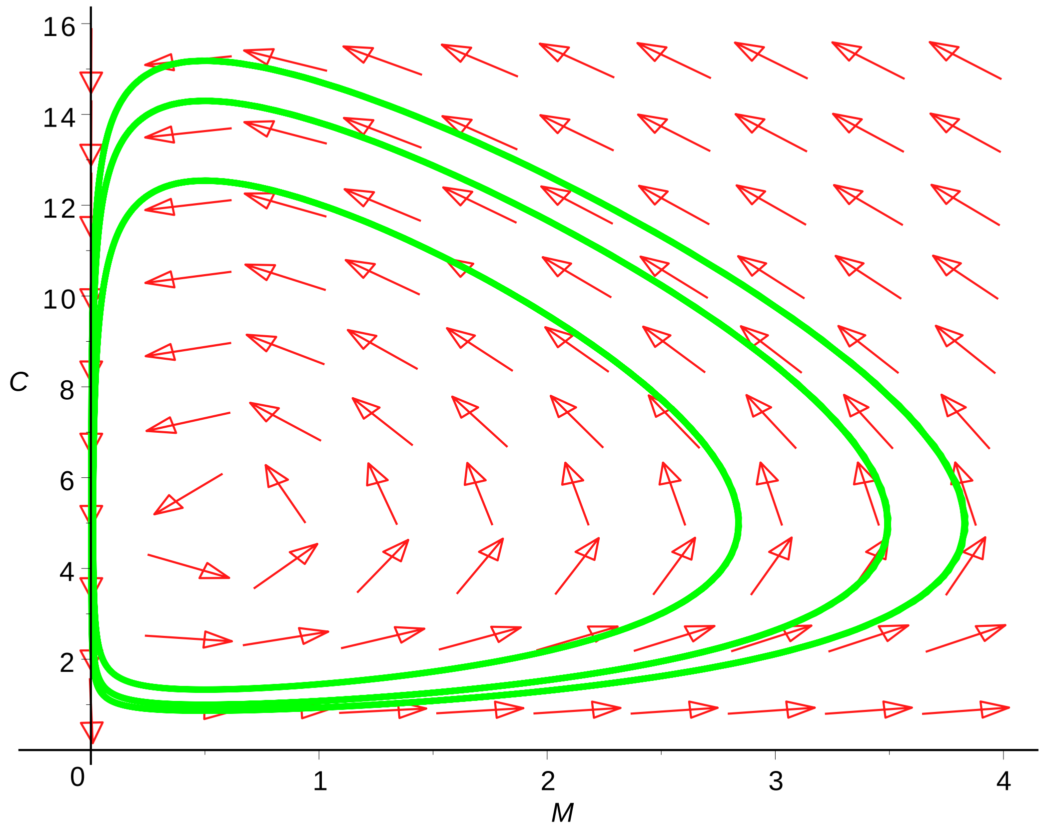
We previously looked at the model "$\frac{dx}{d\Gamma} = \left(Rx\left(1 - \frac{x}{Q}\right) - \left(\frac{x}{1+x^2}\right)\right)$ "(Moody's, pg 279) for the spruce budworm model with predation. Another population model that is presented to us in our text in Chapter 7 is the "Lotka-Volterra"(Greenberg pg 450) model. Here there are two populations

with the predator as" $\frac{dx}{dt} = (\propto - \beta y)x$ & $\frac{dy}{dt} = (-\gamma + \delta x)y$" (Greenberg, pg 450) for the prey.

I happen to have many animals in my home. But, what we have most of in my home is cats. At one point we had 13 cats living in our home. So for fun I am going to look at a mouse and cat model so that : (M= Mouse and C= Cat)

$$\frac{dM}{dt} = 1.0\,M - 0.2\,MC$$
$$\frac{dC}{dt} = 0.4\,MC - 0.2\,C$$

We can then get the following phase line relationship for this model:



So if we continue with the model we can take the Jacobian of the above model as follows:

$$A = \begin{bmatrix} \frac{\partial M'}{\partial M} & \frac{\partial M'}{\partial C} \\ \frac{\partial C'}{\partial M} & \frac{\partial C'}{\partial C} \end{bmatrix} = \begin{bmatrix} 1.0 - 0.2\,C & 0.2\,M \\ 0.4\,C & 0.4\,M - 0.2 \end{bmatrix}$$

If we get the eigenvalue of this matrix we get:

$$0.4000000000 - 0.1000000000\,C + 0.2000000000\,M + 0.1000000000\sqrt{36. - 12.\,C - 24.\,M + C^2 + 12.\,M\,C + 4.\,M^2},\ 0.4000000000 - 0.1000000000\,C + 0.2000000000\,M - 0.1000000000\sqrt{36. - 12.\,C - 24.\,M + C^2 + 12.\,M\,C + 4.\,M^2}$$

We can then test any given point to see if it is stable at that point.We can try the following example:

If we consider when Mice are 4 and cats are 2 we get:

$$\begin{bmatrix} 1.0-0.2\cdot 2 & 0.2\cdot 4 \\ 0.4\cdot 2 & 0.4\cdot 4-0.2 \end{bmatrix}$$

If we get the eigenvalues of this matrix we get:

$0.105572809000084, 1.89442719099992$

Since we have positive eigenvalues so the model is not stable at this point.

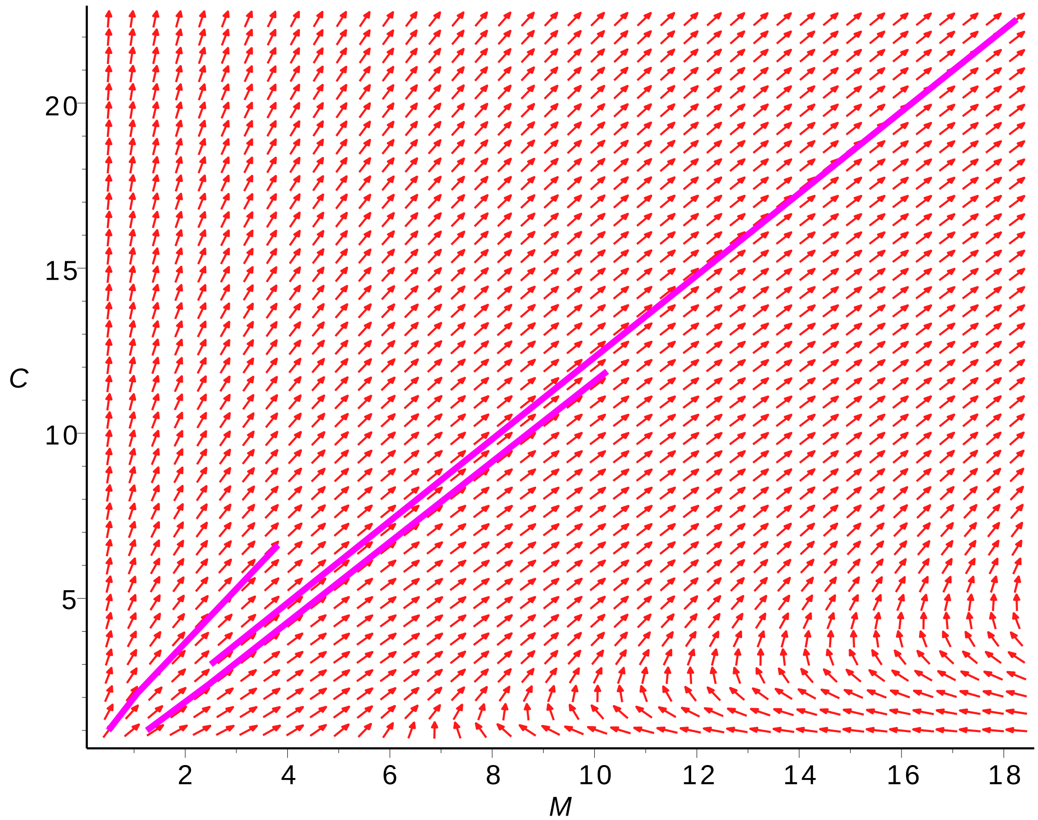
We can also investigate competing species which is according to Greenberg the model is:

" $\frac{dx}{dt}=(a-bx-cy)x$

$\frac{dy}{dt}=(d-ex-fy)y$ "(Greenberg pg 253)

I also have quite a number of chipmunk families living in our front and back yard. So I will let the M= to mouse and C this time equal to chipmunks.

$$\frac{dM}{dt}=4.0\cdot M-1.0\cdot M\cdot M-2.9\cdot C\cdot M$$
$$\frac{dC}{dt}=5.1\cdot C-1.1\,M\cdot C-1.1\cdot C\cdot C$$



So if we continue with the model we can take the Jacobian of the above model as follows:

$$B = \begin{bmatrix} \frac{\partial M'}{\partial M} & \frac{\partial M'}{\partial C} \\ \frac{\partial C'}{\partial M} & \frac{\partial C'}{\partial C} \end{bmatrix} = \begin{bmatrix} 4.0 - 2.0 \cdot M - 2.9 \cdot C & -2.9 \cdot M \\ -1.1 \cdot C & 45.1 - 1.1 \cdot MM - 2.2 \cdot C \end{bmatrix}$$

$$JB := \begin{bmatrix} 4.0 - 2.0 \cdot M - 2.9 \cdot C & -2.9 \cdot M \\ -1.1 \cdot C & 45.1 - 1.1 \cdot MM - 2.2 \cdot C \end{bmatrix}:$$

If we get the eigenvalue of this matrix we get:

$$24.55000000 - 1.\, M - 2.550000000\, C - 0.5500000000\, MM$$
$$+ 0.05000000000\, \big(1.68921\, 10^5 + 16440.\, M + 5754.\, C - 9042.\, MM + 400.\, M^2 + 1556.\, M\, C$$

$$- 440.\, M\, MM + 49.\, C^2 - 154.\, C\, MM + 121.\, MM^2 \big)^{1/2},\ 24.55000000 - 1.\, M$$
$$- 2.550000000\, C - 0.5500000000\, MM$$
$$- 0.05000000000\, \big(1.68921\, 10^5 + 16440.\, M + 5754.\, C - 9042.\, MM + 400.\, M^2 + 1556.\, M\, C$$
$$- 440.\, M\, MM + 49.\, C^2 - 154.\, C\, MM + 121.\, MM^2 \big)^{1/2}$$

We can then test any given point to see if it is stable at that point. We can try the following example:

If we consider when Mice are 12 and Chipmunks are 12 we get:

$$\begin{bmatrix} 4.0 - 2.0\cdot 12 - 2.9\cdot 14 & -2.9\cdot 12 \\ -1.1\cdot 14 & 45.1 - 1.1\cdot 12\cdot 12 - 2.2\cdot 14 \end{bmatrix}$$

If we get the eigenvalues of this matrix we get:

$$-54.6113102400160, -150.088689759984$$

Here we have all negative eigenvalues so the system is stable at this point.

According to Strogatz many mathematical biologists do not use the Lotka Voltera model because it depicts a continual array of stable cycles for a relationship and there are upperboundaries that populations can approach.. Although this may be true, it still can be helpful in gauge in certain relationships that are tightly intertwined like the cheetah and gazelles. (Although my cats probably think of themselves as cheetahs, they always come home for breakfast and dinner.)

The more advanced population models that are used by many mathematical biologists will include factors such as "aging, growth rates, diffusion, taxis & mortality."(Magal 2008) The perspective gets more complex when one considers population models from an epidemiological perspective with host-parasite models. Usually with the models there is a large increase or spike in the model when there is an outbreak, followed by a decrease as the parasite is unable to spread at an increasing rate (people who were exposed and now have immunity and those who have died as well are now removed from potential hosts). While one has an increase in there is an epidemic and as it starts to decrease there is no longer an epidemic. The spruce budworm model closely resembles this pattern in that it increases as spruce trees and other trees it likes are lush and abundant. As there are damaged trees and dead trees in a location the outbreak tends to level of and decrease and there is no longer an outbreak. Knowing when these Spruce trees will be in their prime state when restoring forests can help officials anticipate favourable environments for the spruce budworm and take preventative measures.

Spruce Trees

The Spruce Budworm outbreak issue is a complex issue and the other side of what is going on here is with the trees they thrive from. There are five main types of spruce trees in North America; White, Black, Englemann, Sitka & Red. The Red spruce is the prevalent one throughout the southern half of Quebec, New Brunswick, Nova Scotia, & the New England States including the Adirondacks of New York.

In North America there is a tremendous wood harvesting that takes place for human consumption/economic needs. As part of this industry there are various replanting strategies used. According to Grossnickle in 1994 "695MM seedlings" were planted with "436MM" of them being spruce. This is important since reforestation efforts are important not just to industry but as a food base for the spruce budworm. Despite the name Spruce Budworm, it does not only feed on Spruce trees. I bring this up because the introduction of new tree species in reforestation or tree farms will at times contribute to outbreaks as well. Another example of this according to Wainhouse is with the Southern US where in certain regions the dominant tree was the longleaf pine. Reforestation efforts took place with the introduced species the lobloly pine. It turned out that the introduced species was not as resistant to the pine beetle and as a result there was dramatic increase in the pine beatle population. A similar situation has occurred in certain regions where new tree species were introduced during reforestation and farming in the northern part of the US and Canada and the result has been in some of these regions an increase in the Spruce Budworm populations since the introduced species are not as resilient to the Spruce Budworms.

Modeling the activity of farming new trees in a location can be key to knowing when there will be a lush food source for the Budworms. One action that can be done is to continue pursuing the usage of biological insecticides. At the moment there is apparently limited success using such agents to kill the eggs of the Budworm. Perhaps we can genetically modify these agents to better target the eggs and not affect the seeds of new Spruce trees or the fungi that help assist the Spruce trees absorb nutrients and grow healthy. Doing so may be complex pursuit but, it the benefits is that there would not be toxic affects to the environment etc, which would prove helpful for all people as well. While chemical insecticides are extremely effective the side effect is that the side effect is that there are harmful chemicals added to the environment.(Which includes our drinking water.)

Studying these models can hopefully reveal what conditions cause outbreaks in different species to occur. For our planet (earth) in most cases we would want to keep these conditions so that we maintain a steady state and there are not significant imbalances causing ecological and economic losses. For scientists who are preparing for our next leap as a space faring species, they would want steady states for any ships, but may want to cause outbreaks in other situations such as an orchestrated terraforming of Mars(they would want to send enough micro organisms on the initial trip to cause an outbreak based on the conditions existing there.) Taking this a step further, we would want to understand how genetically modified organisms will behave with a modified trait as well. Whatever the purpose or motive, greater understanding how different living organisms are likely to behave will be important to us as a species.

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