Snowflakes

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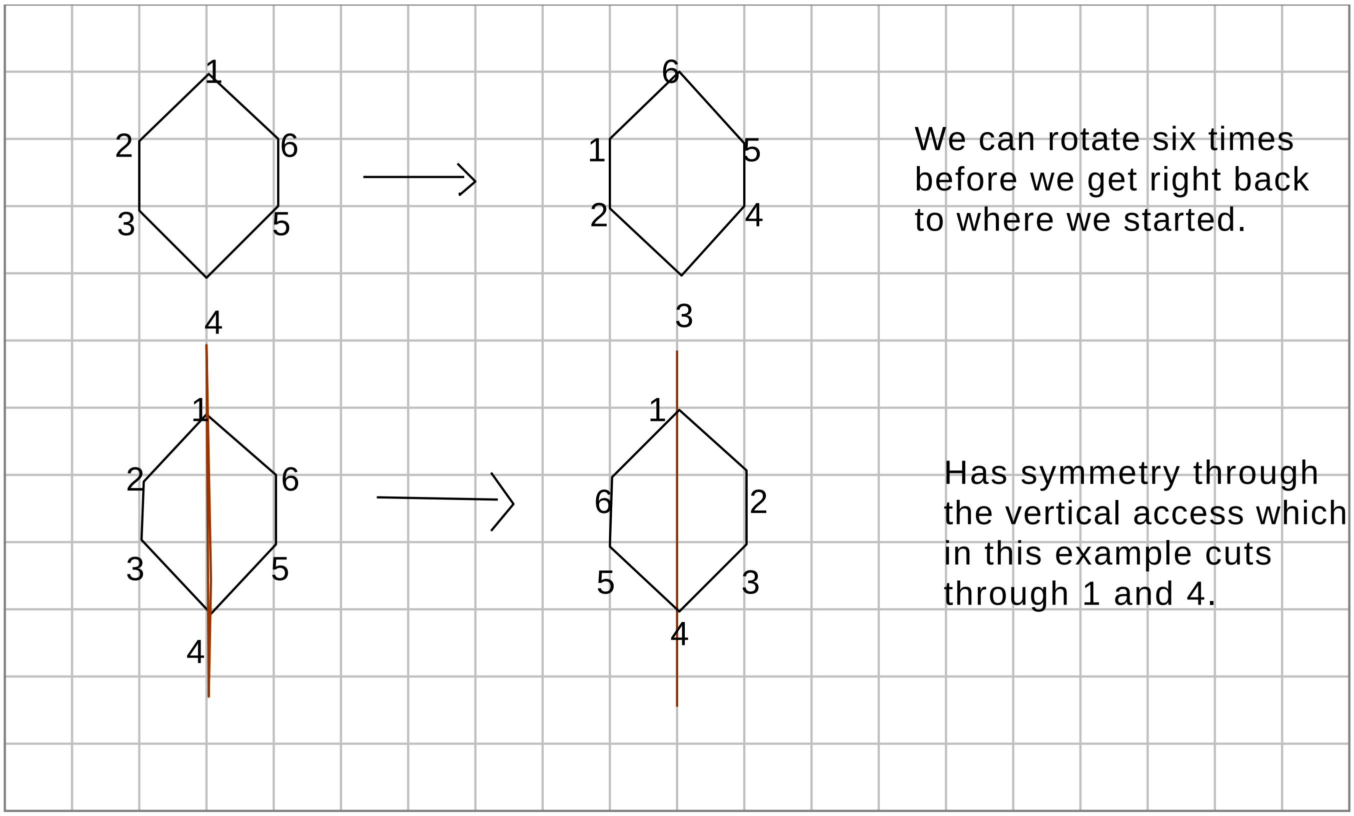
September 29, 2013

The site of snowflakes is another beautiful example of how mathematics is all around us. What nature often has in common with mathematics is that one can apply group theory to the underlying structure. One such example that touches many lives with beautiful memories of skiing is snowflakes. While not everyone lives where there is snow during the year, most people at some point or another have seen snow in their lives. Even if you have never seen it in person, then chances are you saw it in a movie or on a television show. It turns out that the way the water crystallizes, it usually has six sides to it. The D_{12} dihedral group is an excellent way to represent this six sided structure.
Some nice sites on the shapes of snowflakes are:
http://siarchives.si.edu/sites/default/files/pdfs/1.%20Looking%20at%20Snowflakes%20-%20Shape%20and%20Structure%20Activity.pdf
http://bentley.sciencebuff.org/index.html

Since snowflakes crystallize in the shape of a hexagonal prism. If we use the dihedral group D_{12} to represent this structure then we get a set of 12 elements. We can look at this in detail and consider the rotations and the symmetry. The rotations alone will be $\{1, r, r^2, r^3, r^4, r^5\}$. If we consider symmetry then we also have $\{s, sr, sr^2, sr^3, sr^4, sr^5\}$. If we consider both the rotations and the symmetry we have the twelve elements $\{1, r, r^2, r^3, r^4, r^5, s, sr, sr^2, sr^3, sr^4, sr^5\}$ in this set. We can also state that this group is non-abelian such that $a \star b \neq b \star a$ for all the elements in the group. $(i.e.\ sr \neq rs)$



While this example with snowflakes is an example of how group theory can be applied to something that many people are familiar with each winter there are countless applications to group theory. If we extend out from our example here with water, which is made from the chemicals hydrogen and oxygen, group theory is also used by chemists in how various chemical structures can form. Group theory is also used by fields such as music and cryptography. The applications of group theory seem as limitless as our imagination.

References

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