How to use the quadratic equation to find the two binomials of a quadratic equation

Joseph Pousada

May 29, 2017

1 Factoring

In this example we will look at a quadratic equation and factor. In addition, we will look at using the quadratic equation to find the two binomials as well for this quadratic equation.

Factoring

Here we are going to look at the following quadratic equation and look at a technique for factoring:

$$12x^2 + 11x - 5$$

Note quadratic equations are in the form $ax^2 + bx + c$ where $a, b \,\&\, c$ are scalars. So, in the above equation we will first consider the a or 12 in front of the x^2 and the c or the -5. We start by multiplying the scalars a & b which in this example is $12 * (-5)$ gives us -60. We then consider various multiples of -60 and decide which multiples when added will give us the b scalar. So in our case here which multiples of -60 will add up to $+11$. Here I have a list of multiples of 60 for us to consider:

$$(-60) * 1$$
$$(-1) * 60$$
$$2 * (-30)$$
$$(-2) * 30$$
$$4 * (-15)$$
$$(-4) * 15$$

If we consider the last multiple we can see that $(-4) * 15 = (-60)$ and that $(-4) + (15) = 11$. We can use this knowledge to expand our quadratic equation as such:

$$12x^2 - 4x + 15x - 5$$

We can now group the elements as such:

$$(12x^2 - 4x)(+15x - 5)$$

From the first group on the left we can factor out a $(4x)$:

$$4x(3x-1)(+15x-5)$$

From the second group on the right we can factor out a $(+5)$:

$$4x(3x-1)+5(3x-1)$$

Now we can factor out a $(3x-1)$ so that we get:

$$(4x+5)(3x-1)$$

and we have successfully factored our quadratic equation.

We were not asked to find the roots but if we were asked to find the roots such that $12x^2+11x-5=0$ we can factor to get:

$$(4x+5)(3x-1)=0$$

We would then consider when each of these binomials equals zero. Considering the first binomial we would get:

$$(4x+5)=0$$

Subtracting 5 from both sides we get:

$$4x+5-5=0-5$$

Simplifying we get:

$$4x=-5$$

Dividing both sides by 4 we get:

$$\frac{4x}{4}=\frac{-5}{4}$$

Simplifying we get:

$$x=\frac{-5}{4}$$

We have our first root. We can then consider when our second binomial is equal to zero:

$$(3x-1)=0$$

Adding 1 to both sides we get:

$$3x-1+1=0+1$$

Simplifying we get:

$$3x=1$$

Dividing both sides by 3 we get:

$$\frac{3x}{3}=\frac{1}{3}$$

Simplifying we get:

$$x=\frac{1}{3}$$

We now have our second root.

2 Using the Quadratic Equation

Here we will look at using the quadratic equation to find the same binomial of:

$$12x^2 + 11x - 5$$

Again, please note that quadratic equations are in the form $ax^2 + bx + c$ where $a, b \,\&\, c$ are scalars. We can see here that our $a = 12$, $b = 11 \,\&\, c = (-5)$ We now consider the quadratic equation which is:

$$x = \frac{-b \pm \sqrt{-b^2 - 4ac}}{2a}$$

We can substitute our values for $a, b \,\&\, c$ to get:

$$x = \frac{-11 \pm \sqrt{-11^2 - 4(12)(-5)}}{2(12)}$$

We can then simplify to get:

$$x = \frac{-11 \pm \sqrt{121 + 240}}{24}$$

We then add our values under the square root to get:

$$x = \frac{-11 \pm \sqrt{361}}{24}$$

Taking the square root of 361 we simplify this to:

$$x = \frac{-11 \pm 19}{24}$$

Considering the $\pm$ symbol we now split this into two parts, one with the $+$ operation and the next with the $-$ operation. Considering the $+$ operation we get:

$$x = \frac{-11 + 19}{24}$$

If we simplify our numerator we get:

$$x = \frac{8}{24}$$

If we simplify our fraction we get:

$$x = \frac{1}{3}$$

We now will consider the $-$ operation:

$$x = \frac{-11 - 19}{24}$$

If we simplify our numerator we get:

$$x = \frac{-30}{24}$$

If we simplify our fraction we get:

$$x = \frac{-5}{4}$$

The quadratic equation is set to give us the roots of the quadratic equation and considers when $ax^2 + bx + c = 0$. However, we can work backwards since we already have our roots. Considering the first root we get:

$$x = \frac{1}{3}$$

We can multiply both sides by 3:

$$(3) * x = \frac{1}{3} * (3)$$

Once we simplify we get:

$$3x = 1$$

We can now subtract one from both sides:

$$3x - 1 = 1 - 1$$

Once we simplify we get:

$$3x - 1 = 0$$

We now know that one of our binomials will be $(3x - 1)$. Doing this again for the second root we get:

$$x = \frac{-5}{4}$$

We can multiply both sides by 4 to get:

$$(4)x = \frac{-5}{4}(4)$$

Once we simplify we get:

$$4x = -5$$

We can now add 5 to both sides:

$$4x + 5 = -5 + 5$$

Once we simplify we get:

$$4x + 5 = 0$$

We now know that one of our binomials will be $(4x + 5)$.

Therefore, we now have the two binomials and can conclude that if we factor the above quadratic equation the factored binomials will be:

$$(3x - 1)(4x + 5)$$

When asked to find the factor of a binomial it is useful to use the various factoring techniques that your math teacher has taught you since it is both time efficient and requires less work. However, if you are stuck then you may wish to use the quadratic method. Since this is time consuming if you are using this on a test I would suggest marking the question for review and coming back to the question once you are done with your exam to rework this question. Once you come back to the question you may want to spend up to a minute to see if your factoring techniques work. If you are still stuck and want to pursue the answer then use the quadratic formula and work backwards to get the binomial factors as shown. I hope this proves helpful.