Proof That A Group Of Order 5313 Is Not Simple.

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January 5th, 2014

We will prove by way of contradiction that a group of order 5313 is not simple. First, we let G be a group and let us assume $|G| = 5313$. We will consider the prime factorization of this group is $(3)\ (7)\ (11)\ (23)$. We will now apply Sylow's theorem to the group G .

If we start with n_{23} then per Sylow's theorem :

$n_{23} | (3 \cdot 7 \cdot 11)\ \ n_{23} \equiv 1 (\mathbf{mod}\ 23) \Rightarrow n_{23} = 1$ or $n_{23} = 231$

If we assume that $n_{23} \neq 1$ then the number of elements of order 23 are $22 \cdot 231 = 5082$.

If we now consider n_{11} then per Sylow's theorem

$n_{11} | (3 \cdot 7 \cdot 23)\ \ n_{11} \equiv 1 (\mathbf{mod}\ 11) \Rightarrow n_{11} = 1$ or $n_{11} = 23$

If we assume that $n_{11} \neq 1$ then the number of elements of order 11 are $10 \cdot 23 = 230$.

If we now consider n_7 then per Sylow's theorem

$n_7 | (3 \cdot 11 \cdot 23)\ \ n_7 \equiv 1 (\mathbf{mod}\ 7) \Rightarrow n_7 = 1$ or $n_7 = 253$

If we assume that $n_7 \neq 1$ then the number of elements of order 7 are $6 \cdot 253 = 1518$.

We do not need to even consider n_3 at this point since the total number of elements with $n_{23} \neq 1$, $n_{11} \neq 1$, $n_7 \neq 1$ or $n_{23} = 5082$, $n_{11} = 230$ & $n_7 = 1518$ is 5082 + 1518 = 6830 > $|G|$.

Therefore, we conclude that G must not be simple and must have a normal Sylow subgroup.

References

Dummit, D. S., & Foote, R. M. (2004). Abstract algebra. Hoboken, NJ: Wiley.
The Integers 1 to 10000, in groups of 100. (n.d.). Retrieved from http://www.positiveintegers.org/IntegerGroups/1-10000