

Inverse Matrices

If you are taking a class in Linear Algebra you may have to manually derive an inverse matrix. The method for doing so manually is quite simple. You take the matrix you want to get the inverse of and augment it with the same size identity matrix. You then proceed to the the reduce row echelon form of the original matrix. Doing so will convert the augmented identity matrix and leave you with the inverse matrix on the right hand side of the original matrix. Here is an example using a randomly generated matrix in Maple. The Maple package used in this example is "linalg" and the key commands used are "augment" to augment the original matrix with the identity matrix, "mulrow" to multiply a specific row by an assigned value of your choice and "addrow" to add one row to another assigned row by a specific assigned quantity.

restart :

with(linalg) :

$$\mathbf{A} := \begin{bmatrix} -93 & -32 & 8 & 44 \\ -76 & -74 & 69 & 92 \\ -72 & -4 & 99 & -31 \\ -2 & 27 & 29 & 67 \end{bmatrix} :$$

$$\mathbf{I}_4 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} :$$

$\mathbf{AI} := \text{augment}(\mathbf{A}, \mathbf{I}_4);$

print(); # input placeholder

$$\begin{bmatrix} -93 & -32 & 8 & 44 & 1 & 0 & 0 & 0 \\ -76 & -74 & 69 & 92 & 0 & 1 & 0 & 0 \\ -72 & -4 & 99 & -31 & 0 & 0 & 1 & 0 \\ -2 & 27 & 29 & 67 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(1.1)

$\mathbf{AI} := \text{mulrow}\left(\mathbf{AI}, 1, -\frac{1}{93}\right);$

print(); # input placeholder

$$\begin{bmatrix}
 1 & \frac{32}{93} & -\frac{8}{93} & -\frac{44}{93} & -\frac{1}{93} & 0 & 0 & 0 \\
 -76 & -74 & 69 & 92 & 0 & 1 & 0 & 0 \\
 -72 & -4 & 99 & -31 & 0 & 0 & 1 & 0 \\
 -2 & 27 & 29 & 67 & 0 & 0 & 0 & 1
 \end{bmatrix} \tag{1.2}$$

AI := addrow(AI, 1, 2, 76);

print(); # input placeholder

$$\begin{bmatrix}
 1 & \frac{32}{93} & -\frac{8}{93} & -\frac{44}{93} & -\frac{1}{93} & 0 & 0 & 0 \\
 0 & -\frac{4450}{93} & \frac{5809}{93} & \frac{5212}{93} & -\frac{76}{93} & 1 & 0 & 0 \\
 -72 & -4 & 99 & -31 & 0 & 0 & 1 & 0 \\
 -2 & 27 & 29 & 67 & 0 & 0 & 0 & 1
 \end{bmatrix} \tag{1.3}$$

AI := addrow(AI, 1, 3, 72);

print(); # input placeholder

$$\begin{bmatrix}
 1 & \frac{32}{93} & -\frac{8}{93} & -\frac{44}{93} & -\frac{1}{93} & 0 & 0 & 0 \\
 0 & -\frac{4450}{93} & \frac{5809}{93} & \frac{5212}{93} & -\frac{76}{93} & 1 & 0 & 0 \\
 0 & \frac{644}{31} & \frac{2877}{31} & -\frac{2017}{31} & -\frac{24}{31} & 0 & 1 & 0 \\
 -2 & 27 & 29 & 67 & 0 & 0 & 0 & 1
 \end{bmatrix} \tag{1.4}$$

AI := addrow(AI, 1, 4, 2);

print(); # input placeholder

$$\begin{bmatrix}
 1 & \frac{32}{93} & -\frac{8}{93} & -\frac{44}{93} & -\frac{1}{93} & 0 & 0 & 0 \\
 0 & -\frac{4450}{93} & \frac{5809}{93} & \frac{5212}{93} & -\frac{76}{93} & 1 & 0 & 0 \\
 0 & \frac{644}{31} & \frac{2877}{31} & -\frac{2017}{31} & -\frac{24}{31} & 0 & 1 & 0 \\
 0 & \frac{2575}{93} & \frac{2681}{93} & \frac{6143}{93} & -\frac{2}{93} & 0 & 0 & 1
 \end{bmatrix} \tag{1.5}$$

AI := mulrow(AI, 2, $-\frac{93}{4450}$);

print(); # input placeholder

$$\begin{bmatrix}
 1 & \frac{32}{93} & -\frac{8}{93} & -\frac{44}{93} & -\frac{1}{93} & 0 & 0 & 0 \\
 0 & 1 & -\frac{5809}{4450} & -\frac{2606}{2225} & \frac{38}{2225} & -\frac{93}{4450} & 0 & 0 \\
 0 & \frac{644}{31} & \frac{2877}{31} & -\frac{2017}{31} & -\frac{24}{31} & 0 & 1 & 0 \\
 0 & \frac{2575}{93} & \frac{2681}{93} & \frac{6143}{93} & -\frac{2}{93} & 0 & 0 & 1
 \end{bmatrix} \tag{1.6}$$

$$\text{AI} := \text{addrow} \left(\text{AI}, 2, 3, -\frac{644}{31} \right);$$

print(); # input placeholder

$$\begin{bmatrix}
 1 & \frac{32}{93} & -\frac{8}{93} & -\frac{44}{93} & -\frac{1}{93} & 0 & 0 & 0 \\
 0 & 1 & -\frac{5809}{4450} & -\frac{2606}{2225} & \frac{38}{2225} & -\frac{93}{4450} & 0 & 0 \\
 0 & 0 & \frac{266833}{2225} & -\frac{90631}{2225} & -\frac{2512}{2225} & \frac{966}{2225} & 1 & 0 \\
 0 & \frac{2575}{93} & \frac{2681}{93} & \frac{6143}{93} & -\frac{2}{93} & 0 & 0 & 1
 \end{bmatrix} \tag{1.7}$$

$$\text{AI} := \text{addrow} \left(\text{AI}, 2, 4, -\frac{2575}{93} \right);$$

print(); # input placeholder

$$\begin{bmatrix}
 1 & \frac{32}{93} & -\frac{8}{93} & -\frac{44}{93} & -\frac{1}{93} & 0 & 0 & 0 \\
 0 & 1 & -\frac{5809}{4450} & -\frac{2606}{2225} & \frac{38}{2225} & -\frac{93}{4450} & 0 & 0 \\
 0 & 0 & \frac{266833}{2225} & -\frac{90631}{2225} & -\frac{2512}{2225} & \frac{966}{2225} & 1 & 0 \\
 0 & 0 & \frac{11565}{178} & \frac{8765}{89} & -\frac{44}{89} & \frac{103}{178} & 0 & 1
 \end{bmatrix} \tag{1.8}$$

AI := mulrow $\left(\text{AI}, 3, \frac{2225}{266833} \right);$

print(); # input placeholder

$$\begin{bmatrix}
 1 & \frac{32}{93} & -\frac{8}{93} & -\frac{44}{93} & -\frac{1}{93} & 0 & 0 & 0 \\
 0 & 1 & -\frac{5809}{4450} & -\frac{2606}{2225} & \frac{38}{2225} & -\frac{93}{4450} & 0 & 0 \\
 0 & 0 & 1 & -\frac{90631}{266833} & -\frac{2512}{266833} & \frac{138}{38119} & \frac{2225}{266833} & 0 \\
 0 & 0 & \frac{11565}{178} & \frac{8765}{89} & -\frac{44}{89} & \frac{103}{178} & 0 & 1
 \end{bmatrix} \tag{1.9}$$

AI := addrow $\left(\text{AI}, 3, 4, -\frac{11565}{178} \right);$

print(); # input placeholder

$$\left[\left[1, \frac{32}{93}, -\frac{8}{93}, -\frac{44}{93}, -\frac{1}{93}, 0, 0, 0 \right], \right. \quad (1.10)$$

$$\left[0, 1, -\frac{5809}{4450}, -\frac{2606}{2225}, \frac{38}{2225}, -\frac{93}{4450}, 0, 0 \right],$$

$$\left[0, 0, 1, -\frac{90631}{266833}, -\frac{2512}{266833}, \frac{138}{38119}, \frac{2225}{266833}, 0 \right],$$

$$\left[0, 0, 0, \frac{64334045}{533666}, \frac{31292}{266833}, \frac{26183}{76238}, -\frac{289125}{533666}, 1 \right] \right]$$

$$\text{AI} := \text{mulrow} \left(\text{AI}, 4, \frac{533666}{64334045} \right);$$

print(); # input placeholder

$$\left[\left[1, \frac{32}{93}, -\frac{8}{93}, -\frac{44}{93}, -\frac{1}{93}, 0, 0, 0 \right], \right. \quad (1.11)$$

$$\left[0, 1, -\frac{5809}{4450}, -\frac{2606}{2225}, \frac{38}{2225}, -\frac{93}{4450}, 0, 0 \right],$$

$$\left[0, 0, 1, -\frac{90631}{266833}, -\frac{2512}{266833}, \frac{138}{38119}, \frac{2225}{266833}, 0 \right],$$

$$\left[0, 0, 0, 1, \frac{62584}{64334045}, \frac{183281}{64334045}, -\frac{57825}{12866809}, \frac{533666}{64334045} \right] \right]$$

$$\text{AI} := \text{addrow} \left(\text{AI}, 2, 1, -\frac{32}{93} \right);$$

print(); # input placeholder

$$\left[\left[1, 0, \frac{808}{2225}, -\frac{156}{2225}, -\frac{37}{2225}, \frac{16}{2225}, 0, 0 \right], \right. \quad (1.12)$$

$$\left[0, 1, -\frac{5809}{4450}, -\frac{2606}{2225}, \frac{38}{2225}, -\frac{93}{4450}, 0, 0 \right],$$

$$\left[0, 0, 1, -\frac{90631}{266833}, -\frac{2512}{266833}, \frac{138}{38119}, \frac{2225}{266833}, 0 \right],$$

$$\left[0, 0, 0, 1, \frac{62584}{64334045}, \frac{183281}{64334045}, -\frac{57825}{12866809}, \frac{533666}{64334045} \right] \Bigg]$$

$$\text{AI} := \text{addrow} \left(\text{AI}, 3, 2, \frac{5809}{4450} \right);$$

print(); # input placeholder

$$\left[\left[1, 0, \frac{808}{2225}, -\frac{156}{2225}, -\frac{37}{2225}, \frac{16}{2225}, 0, 0 \right], \right. \quad (1.13)$$

$$\left[0, 1, 0, -\frac{861667}{533666}, \frac{1278}{266833}, -\frac{1233}{76238}, \frac{5809}{533666}, 0 \right],$$

$$\left[0, 0, 1, -\frac{90631}{266833}, -\frac{2512}{266833}, \frac{138}{38119}, \frac{2225}{266833}, 0 \right],$$

$$\left[0, 0, 0, 1, \frac{62584}{64334045}, \frac{183281}{64334045}, -\frac{57825}{12866809}, \frac{533666}{64334045} \right] \Bigg]$$

$$\text{AI} := \text{addrow} \left(\text{AI}, 3, 1, -\frac{808}{2225} \right);$$

print(); # input placeholder

$$\left[\left[1, 0, 0, \frac{14204}{266833}, -\frac{3525}{266833}, \frac{224}{38119}, -\frac{808}{266833}, 0 \right], \right. \quad (1.14)$$

$$\left[0, 1, 0, -\frac{861667}{533666}, \frac{1278}{266833}, -\frac{1233}{76238}, \frac{5809}{533666}, 0 \right],$$

$$\left[0, 0, 1, -\frac{90631}{266833}, -\frac{2512}{266833}, \frac{138}{38119}, \frac{2225}{266833}, 0 \right],$$

$$\left[0, 0, 0, 1, \frac{62584}{64334045}, \frac{183281}{64334045}, -\frac{57825}{12866809}, \frac{533666}{64334045} \right]$$

$$\text{AI} := \text{addrow} \left(\text{AI}, 4, 3, \frac{90631}{266833} \right);$$

print(); # input placeholder

$$\left[\left[1, 0, 0, \frac{14204}{266833}, -\frac{3525}{266833}, \frac{224}{38119}, -\frac{808}{266833}, 0 \right], \right. \quad (1.15)$$

$$\left[0, 1, 0, -\frac{861667}{533666}, \frac{1278}{266833}, -\frac{1233}{76238}, \frac{5809}{533666}, 0 \right],$$

$$\left[0, 0, 1, 0, -\frac{584392}{64334045}, \frac{295157}{64334045}, \frac{87650}{12866809}, \frac{181262}{64334045} \right],$$

$$\left[0, 0, 0, 1, \frac{62584}{64334045}, \frac{183281}{64334045}, -\frac{57825}{12866809}, \frac{533666}{64334045} \right]$$

$$\text{AI} := \text{addrow} \left(\text{AI}, 4, 2, \frac{861667}{533666} \right);$$

print(); # input placeholder

$$\left[\left[1, 0, 0, \frac{14204}{266833}, -\frac{3525}{266833}, \frac{224}{38119}, -\frac{808}{266833}, 0 \right], \right. \quad (1.16)$$

$$\left. \left[0, 1, 0, 0, \frac{409178}{64334045}, -\frac{744548}{64334045}, \frac{46691}{12866809}, \frac{861667}{64334045} \right], \right.$$

$$\left. \left[0, 0, 1, 0, -\frac{584392}{64334045}, \frac{295157}{64334045}, \frac{87650}{12866809}, \frac{181262}{64334045} \right], \right.$$

$$\left. \left[0, 0, 0, 1, \frac{62584}{64334045}, \frac{183281}{64334045}, -\frac{57825}{12866809}, \frac{533666}{64334045} \right] \right]$$

$$\mathbf{AI} := \text{addrow} \left(\mathbf{AI}, 4, 1, -\frac{14204}{266833} \right);$$

print(); # input placeholder

$$\left[\left[1, 0, 0, 0, -\frac{853217}{64334045}, \frac{368292}{64334045}, -\frac{35884}{12866809}, -\frac{28408}{64334045} \right], \right. \quad (1.17)$$

$$\left. \left[0, 1, 0, 0, \frac{409178}{64334045}, -\frac{744548}{64334045}, \frac{46691}{12866809}, \frac{861667}{64334045} \right], \right.$$

$$\left. \left[0, 0, 1, 0, -\frac{584392}{64334045}, \frac{295157}{64334045}, \frac{87650}{12866809}, \frac{181262}{64334045} \right], \right.$$

$$\left. \left[0, 0, 0, 1, \frac{62584}{64334045}, \frac{183281}{64334045}, -\frac{57825}{12866809}, \frac{533666}{64334045} \right] \right]$$

We now have the inverse matrix to the right of the reduce row echelon form of A.

To check this we can use the commands in maple to get the inverse of A as well.

Ainversecheck := inverse(A);

print(); # input placeholder

$$\left[\begin{array}{r} \frac{853217}{64334045} \\ \frac{409178}{64334045} \\ \frac{584392}{64334045} \\ \frac{62584}{64334045} \\ \frac{368292}{64334045} \\ \frac{744548}{64334045} \\ \frac{295157}{64334045} \\ \frac{183281}{64334045} \\ \frac{35884}{12866809} \\ \frac{46691}{12866809} \\ \frac{87650}{12866809} \\ \frac{57825}{12866809} \\ \frac{28408}{64334045} \\ \frac{861667}{64334045} \\ \frac{181262}{64334045} \\ \frac{533666}{64334045} \end{array} \right] \quad (1.18)$$

I hope this proves helpful in deriving inverse matrices. Feel free to send any questions or suggestions to joseph_pousada@optonline.net. If you need tutoring please contact me at the above email address as well.