D_{14} *Group*

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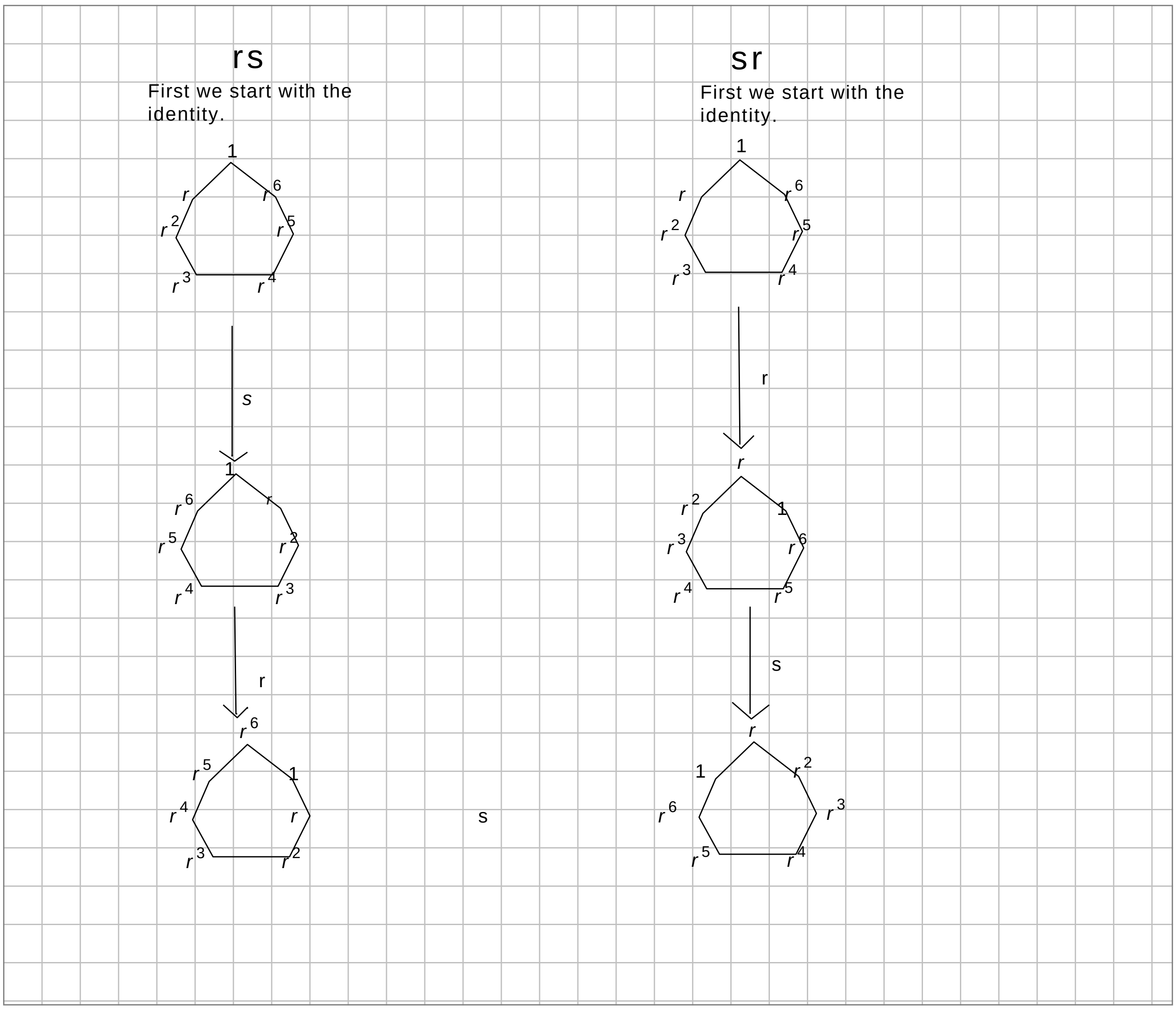
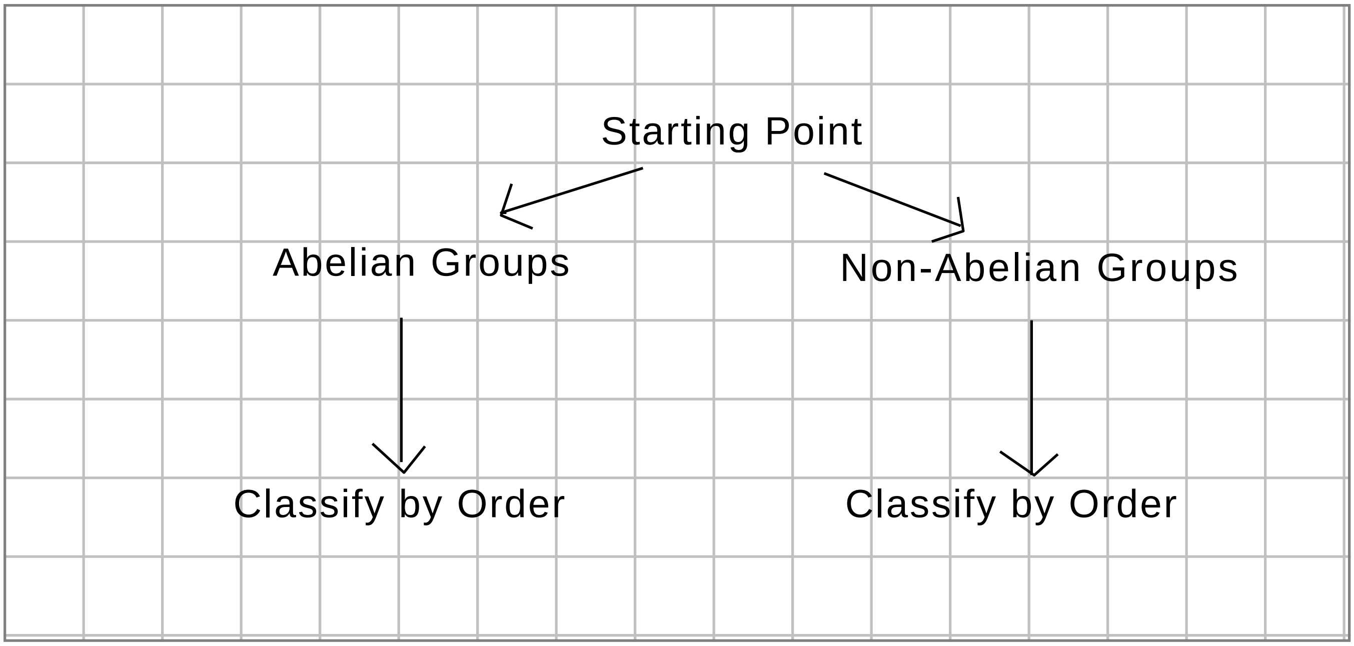
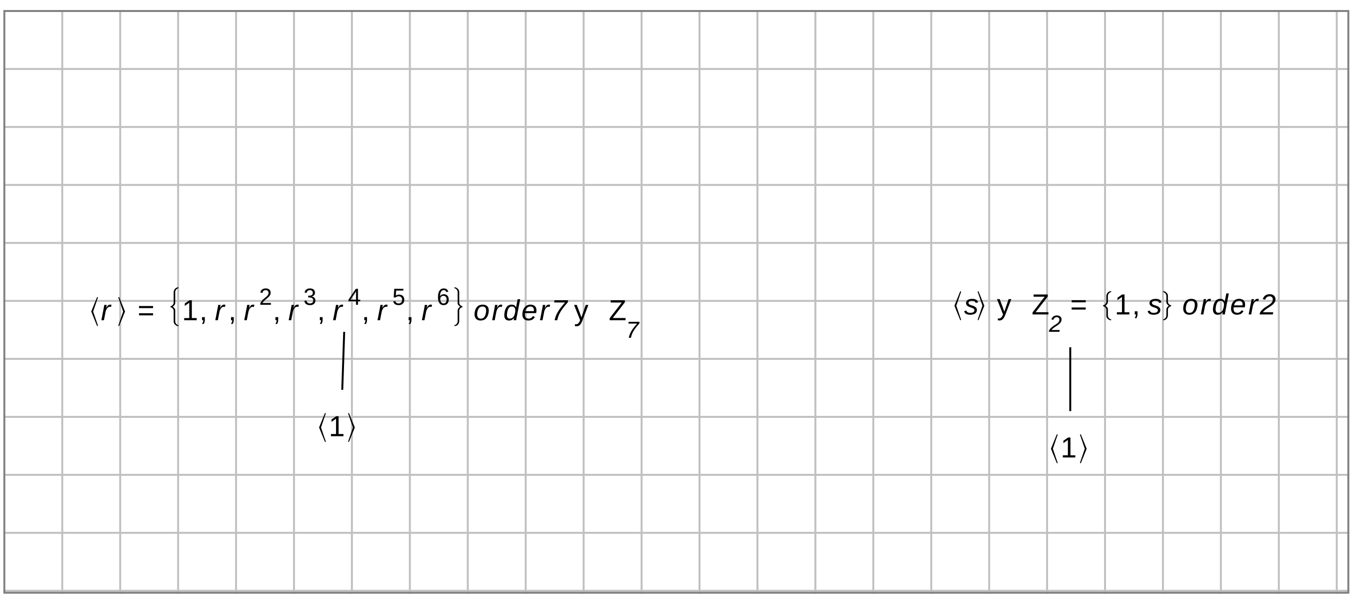
Here we will look at the dihedral group D_{14}. The order of the group is 14, with 14 elements and therefore, it is a finite group. The elements in D_{14} are $\{1, r, r^2, r^3, r^4, r^5, r^6, s, sr, sr^2, sr^3, sr^4, sr^5, sr^6\}$. The generators are r & s . The group is non-abelian which is easily discovered if we consider that $rs \neq sr$. We can demonstrate this visually below:



One approach to group classification is to start by whether they are abelian or non-abelian groups. From there we could classify by order.



Per Sylow's theorem (Dummit, D. S., & Foote, R. M.,pg 93) (2004) D_{14} is of the order $p^{\alpha}m$ where $p = 7, \alpha = 1$ & $m = 2$ & p does not divide m. Therefore D_{14} has a subgroup of order 7. Similarly D_{14} is of the order $p^{\alpha}m$ where $p = 2, \alpha = 1$ & $m = 7$ & p does not divide m. Therefore D_{14} has a subgroup of order 2. The lattice can be seen below:



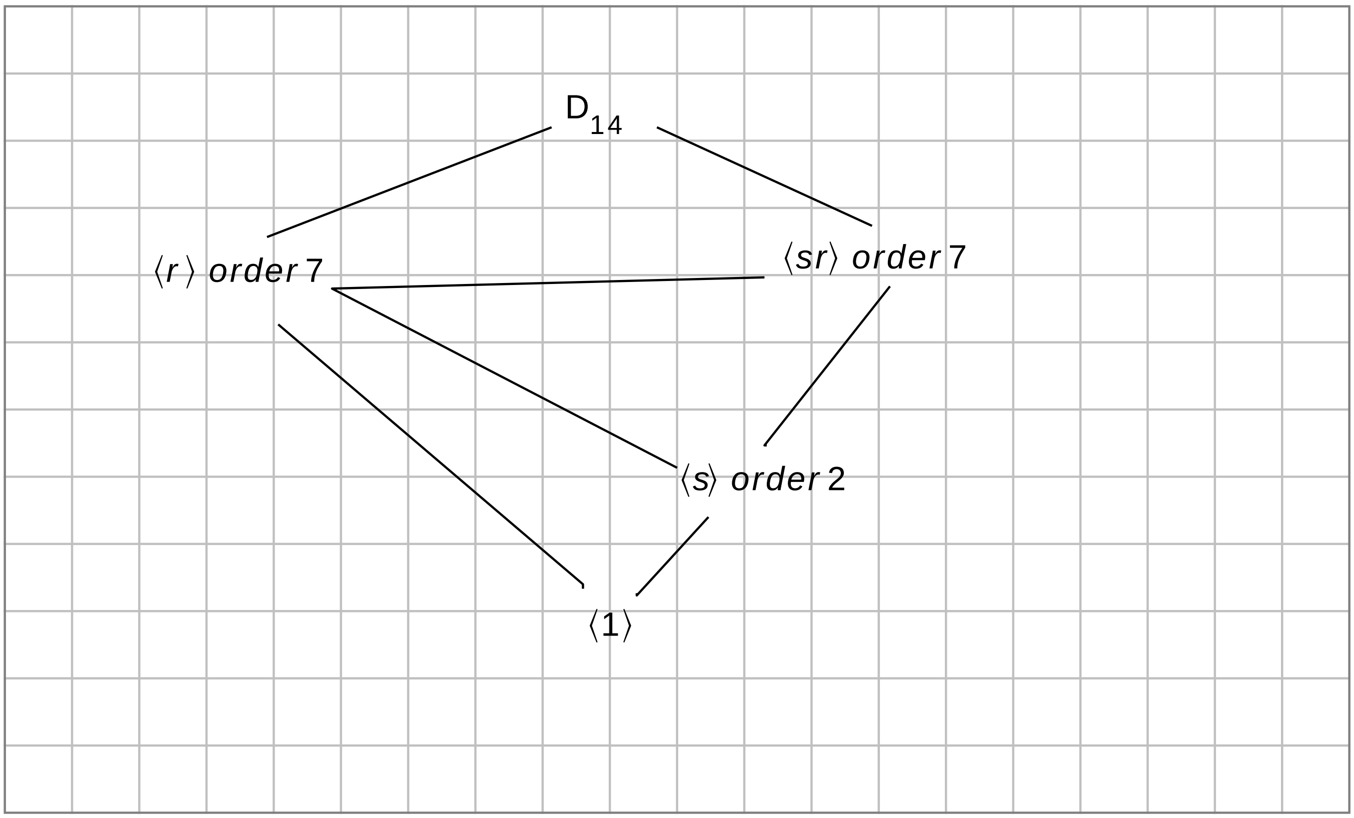
We also know that $\langle r \rangle \cong \mathbb{Z}_7$ & $\langle s \rangle \cong \mathbb{Z}_2$. Sylows theorem also states that there exists a $Sly_p(G)$ subgroup for each prime dividing the order of the group G. By applying this theorem to each of the primes we get:

$n_2 | (7)\ \ n_2 \equiv 1 (\mathbf{mod}\ 2) \Rightarrow n_2 = 1$ or $n_2 = 2$

$n_7 | (2)\ \ n_7 \equiv 1 (\mathbf{mod}\ 7) \Rightarrow n_7 = 1$

By just applying sylows theorem we see that $n_7 = 1$ which implies $Syl_7(G) \trianglelefteq G$. Therefore we conclude that group of order 14 has a normal Sylow $p-subgroup$ for prime 7 dividing its order.

We also know that per theorem 6 (Dummit, D. S., & Foote, R. M. pg 82, (2004)) $N_G(Syl_7) = \mathrm{D}_{14}$, $g(Syl_7)g^{-1} \subseteq (Syl_7)$ & the operation of left cosets makes a group. $\{s, sr, sr^2, sr^3, sr^4, sr^5, sr^6\} = \langle sr \rangle$ of order 7.



Dihedral groups have many applications in robotics. While I am not sure if they used D_{14} in this specific example, Paranjape reports using Dihedrals in robot controls of a flapping wing plane they constructed. One only needs little imagination to see how the Dihedral group and even D_{14} when warranted, could be used in robotics manufacturing.

References:

Dummit, D. S., & Foote, R. M. (2004). Abstract algebra. Hoboken, NJ: Wiley.
Paranjape, A. A. (2013). Novel Dihedral-Based Control of Flapping-Wing Aircraft with Application to Perching. IEEE Transactions on Robotics, 29(5), 1071. Retrieved from

http://arcl.ae.illinois.edu/Paranjape12_perching_submitted.pdf
Dihedral Group -- from Wolfram MathWorld. (n.d.). Retrieved from http://mathworld.wolfram.com/DihedralGroup.html